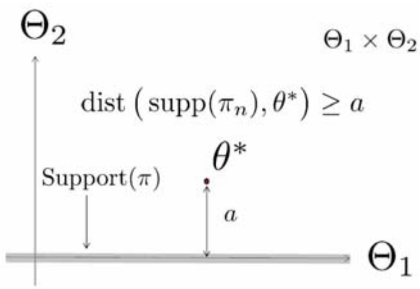
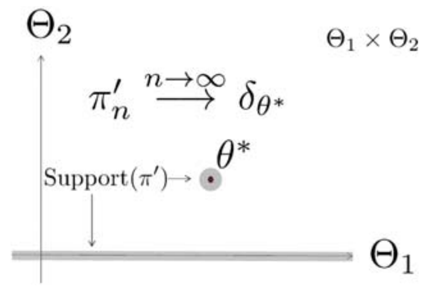
Example 6. Although one may find a prior that is both consistent and *qualitatively robust* when Θ is totally bounded and the model is well-specified, extensions of the mechanism illustrated in Figures 2 and 3 suggest that misspecification implies non *qualitative robustness*. Consider the example illustrated in Figure 4, where the model P is the restriction of a well specified larger model $\bar{P} : \Theta_1 \times \Theta_2 \to \mathcal{M}(X)$ to $\theta_2 = 0$. Assume that the data generating distribution is $\bar{P}(\theta_1^*, \theta_2^*)$ where $\theta_2^* \neq 0$, so that the restricted model P is misspecified. Let π be any prior distribution on $\Theta_1 \times \{\theta_2 = 0\}$. Although π may satisfy Cromwell's rule the mechanisms presented here suggest that is not *qualitatively robust* with respect to perturbed priors having support on $\Theta_1 \times \Theta_2$. Indeed, let π' be an arbitrarily small perturbation of π obtained by removing some mass from the support of π and adding that mass around θ^*. Note that π' can be chosen arbitrarily close to π while satisfying the local consistency assumption, which implies that the posterior distributions of π' concentrate on θ^* while the posterior distributions of π remain supported on $\Theta_1 \times \{\theta_2 = 0\}$. Note that if $\bar{P}$ is interpreted as an extension of the model P, then this mechanism suggests that we can establish conditions under which Bayesian inference is not *qualitatively robust under model extension*.



(a) π



(b) π'

Figure 4: Non-robustness caused by misspecification