Is "Objective Bayesian Analysis" objective, Bayesian, or wise? (Comment on Articles by Berger and by Goldstein)

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"Every form of comfort has its price."

The Eagles

It is a pleasure to have these two papers. Michael Goldstein gives us several examples of the successful use of the subjective approach, and argues that only a subjective approach could be successful with at least some of them. Since Jim Berger does not deny the usefulness of subjective Bayesian analysis – indeed he calls it "indispensable" – I suppose there is hardly anything in Michael's paper that Jim would disagree with, nor would I.

So the issues here have more to do with Jim's paper, in which he argues for a place in the Bayesian repertoire for what he calls objective Bayesian methods. At the outset, it is well to recognize that modeling (for me, this involves both the likelihood and the prior) is approximate, and that one often uses familiar choices with an implicit or explicit hope and expectation that the intended uses are robust against "small" variations in the model. Unlike Jim, I believe that what "small" amounts to in practice will vary by problem (and by analyst), and constitutes part of the judgments we are required to make and defend. Jim would go further and anoint certain of these familiar choices with the label "objective," perhaps in the hopes of deflecting questions about the sensitivity of the conclusions to the judgments embodied in the analysis. It would certainly be nice if a claim of doing objective Bayesian analysis could be sustained. The thesis of this comment is that it cannot.

The name "objective" is not, I think, a good one for advancing the discussion, among other reasons because rhetorically it is opposite of "subjective," which has unfair connotations of incomplete consideration. A better name, I think, would be "interpersonal" or "nonpersonal," to be contrasted to the "personal" philosophy associated with de Finetti, Lindley, Savage, Raiffa, and Ramsey.

Perhaps it would be useful to begin with the question of why personal Bayesian analysis is attractive as a substitute for sampling theory statistics, of either the Fisherian or Neyman-Pearson varieties.

First, the personal view of probability gives reasons why the axioms of probability are what they are, in terms of avoiding sure loss. The usual classical treatments of probability offer no such explanations. The consequences of these axioms are the

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subject of mathematics, but the connection between the axioms and the real world is unaddressed. The frequency interpretation of probability suffers from several deficiencies, notably (1) circularity (you need independent events, defined using probability, to define probability), (2) the judgment of independence is subjective and (3) if you take limiting relative frequency seriously, you know the probabilities of only trivial events, since we lack infinite data sets.

Second, Bayesian analysis contrasts with sampling theory in that, given a model (*i.e.* a likelihood and a prior), the posterior (and optimal decisions, after a utility of loss function is specified) are consequences with no other assumptions made. By contrast, sampling theory has many principles, such as invariance, unbiasedness, maximum likelihood, asymptotics as various parameters approach specifies limits, etc., but no principles of principles, saying which principle to apply when. The extreme of this lack of intellectual coherence was perhaps Fisher's fiducial inference where, faced with certain difficult counterexamples, he finally took the position that the only official fiducial examples are those personally endorsed by Fisher himself.

A third reason for being a Bayesian is the opportunity (and requirement) for a full probabilistic statement of the views of the analyst (both likelihood and prior). Thus rather than attempting a "one-size-fits-all" analysis of a model, regardless of its intended use, personal Bayesians are encouraged (required!) to go deeply into the subject matter of the application. It seems to me that this encourages personal Bayesians to do better science than they might otherwise.

Now let's look at what Jim's Objective Bayesianism implies on each of these matters. He quickly gives up on an explanation of what probability is, for fear of giving offense. Instead it is, for him, simply a mathematical object obeying certain axioms (but why those axioms?).

To deal with the melange of objectivist principles – Jeffrey's priors, reference priors, invariance, maximum-entropy – which contradict each other, Jim proposes to tell us (in Berger et al. (2005)) "unique recommended objective priors," coming from reference priors, invariance considerations and frequentist properties. Shades of R. A. Fisher and fiducialism! Is there a principle here, or only the personal recommendations of a particular group of authors?

Associated with the opportunity-requirement for a full statement of opinion by the analyst goes the taking of responsibility for the assumptions made. To admit that my model is personal means that I must persuade you of the reasonableness of my assumptions in order to convince you to continue reading the work. To claim objectivity is to try to coerce you into consenting, without requiring me to justify the basis for the assumptions. Cloaking these choices in a patina of objectivity does not seems to be a contribution to accurate scientific communication.

So what does objective Bayesianism offer? We are told that it is "the most promising route to the unification of Bayesian and frequentist statistics." The difficulty is that this proposed unification has many of the disadvantages and weaknesses of frequentist statistics, those that make personal Bayesianism attractive in the first place – namely an

explanation of what probability is, a clear and simple theory, and a demand for taking scientific responsibility for the whole model.

Objective Bayesianism is not objective, as Jim admits that the model is subjective. It is not Bayesian, in it violates the likelihood principle (these violations, we are assured, are "minor," but I wonder what analysis supports that claim). But most importantly, it isn't wise, as the price of wrapping oneself in the illusion of objectivity (Berger and Berry (1988)) is much too high.

References

Berger, J., Bernardo, J., and Sun, D. (2005). "Objective Bayesian Inference." Under preparation. 434

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