

within the prescribed area is then given approximately by the term of the Poisson series

$$e^{-m} \frac{m^5}{5!}, \quad (17)$$

or, about 1 in 50,000,000, the probabilities of having 6 or more close neighbours adding very little to this frequency. Since 1500 stars have each this probability of being the centre of such a close cluster of 6, although these probabilities are not strictly independent, the probability that among them any one fulfils the condition cannot be far from and certainly cannot exceed 30 in a million, or 1 in 33,000. Michell arrived at a chance of only 1 in 500,000 but the higher probability obtained by the calculations indicated above is amply low enough to exclude at a high level of significance any theory involving a random distribution.

The force with which such a conclusion is supported is logically that of the simple disjunction: *Either* an exceptionally rare chance has occurred, *or* the theory of random distribution is not true.

In view of the efforts which have been made to force a frequency interpretation on to such a disjunction, it is to be noted that the mental reluctance to accept an event intrinsically improbable would still be felt if, for example, a *datum* were added to Michell's problem to the effect that it was a million to one *a priori* that the stars should be scattered at random. We need not consider what such a statement of probability *a priori* could possibly mean in the astronomical problem; all that is needed is that if this datum were introduced into the calculation, then, in view of the observations, a probability

statement could be inferred *a posteriori*, to the effect that the odds were 30 to 1 that the stars really had been scattered at random. The inherent improbability of what has been observed being observable on this view still remains in our minds, and no explanation has been given of it. It has been overweighted, not neutralized, by the even greater supposed improbability of the universe chosen for examination being of the supposedly exceptional kind in which the stars are *not* distributed at random. The observer is thus not left at all in the same state of mind as if the stars had actually displayed no evidence against a random arrangement, although he would have been forced logically to admit that (so far as statements in terms of probability went) such a theory was probably true, and that the remarkable features that had attracted his attention were, incredible as it might seem, wholly fortuitous.

The example shows that the resistance felt by the normal mind to accepting a story intrinsically too improbable is not capable of finding expression in any calculation of probability *a posteriori*. The variety of ways in which this resistance does express itself very well exhibits its reality. Common reactions are:

- (a) The whole thing is a fabrication.
- (b) There is no sufficient reason to think that the facts were observed and put on record accurately.
- (c) There has been exaggeration, and the omission of circumstances that would help to explain what is claimed.
- (d) Some occult cause, beyond our present understanding, must be invoked.

In the studies known as parapsychology enormous

odds are often claimed, evidently with a view to raising the resistance felt to accepting what is intrinsically improbable to such a pitch that conclusion (d), although itself repugnant, shall be accepted in preference. The incredulous, however, tend to prefer explanations of types (a), (b) or (c) either to accepting such a claim as, let us say, "precognition", or, what seems almost always to be the last choice, to the acceptance as genuine of a very rare contingency.

The fact, important for the understanding of logical situations of this kind, that reluctance to accept a hypothesis strongly contradicted by a test of significance is not removed, though it may be outweighed, by information *a priori*, is exhibited also by the consideration that if the proposed datum, "The odds are a million to one *a priori* that the stars should really be distributed singly and at random"—if this datum were considered as a *hypothesis*, it would be rejected at once by the observations at a level of significance almost as great as the hypothesis, "The stars are really distributed at random", was rejected in the first instance. Were such a conflict of evidence, as has here been imagined under discussion, not in a mathematical department, but in a scientific laboratory, it would, I suggest, be some prior assumption, corresponding to an axiom or a datum in a mathematical argument, that would certainly be impugned.

The attempts that have been made to explain the cogency of tests of significance in scientific research, by reference to supposed frequencies of possible statements, based on them, being right or wrong, thus seem to miss the essential nature of such tests. A man