Logic Primer

1. Argument: a group of statements, one (the conclusion) is claimed to follow from, or be supported by, one or more others (the premises) regarded as providing evidence for the truth of that one.

Note: In a deductive argument, the premises are regarded as providing conclusive evidence for the truth of the conclusion. In inductive arguments, the premises may be regarded as providing good grounds for the truth or approximate truth of the conclusion.

Arguments may be written in the following form:

> premise
> Premise
> …
> ___
> ∴ Conclusion

2. Logic: The study of logic is the study of the methods and principles for distinguishing correct from incorrect arguments.
(In deductive logic, a correct argument is a valid one, and incorrect argument is an invalid one.)

3. Valid (Deductive) Argument: (a) an argument (or argument form) where, if all of the premises are true, then the conclusion must be true.
The following are equivalent definitions of a valid argument and are important to know:
(b) an argument where is (logically) impossible to have all the premises true and the conclusion false.
(c) an argument where, if the conclusion is false, then at least one of the premises must be false.

Notes: Deductive validity is a matter of form—any argument with the same form or pattern as a valid argument is also a valid argument.

4. Invalid (Deductive) Argument: (the denial of a valid argument)
(a) an argument (or argument form) where is possible to have all true premises and a false conclusion.
(b) an argument such that positing all the premises true, and the conclusion false, does not result in a (logical) contradiction.

EXAMPLES:

VALID ARGUMENTS (argument forms):

Modus Ponens (A, B statements)
If A then B.
A.
∴ B

Modus Tollens (A, B statements)
If A then B.
Not-B.
∴Not-A

Disjunctive Syllogism (A, B statements)
Either A or B.
Not A.
∴B.

X, Y classes or properties.

All X's are Y's.	All X's are Y's.
s is an X.	s is not a Y.
∴s is a Y.	∴s is not an X.

Note: In speaking of an argument, the words "true" and "false" apply only to the statements in an argument, not to the argument itself. The words "valid" and "invalid" refer to arguments, but not to statements.

INVALID ARGUMENTS (argument forms):

Affirming the Consequent	**Denying the Antecedent:**
If A then B.	If A then B.
B.	Not-A.
∴ A	∴Not-B

5. Sound (deductive) argument: an argument that is both valid (in form) and has all true premises.

Soundness
Validity/Invalidity and Soundness: Review

I. A valid argument refers to the form of the argument.
Any particular argument that follows or instantiates a valid form is itself valid, regardless of its subject matter. For example, *modus tollens* is a valid argument form:

1. If A then B
2. Not-B

Therefore, not-A

A and B represent sentences. To prove this is valid, we show that if we assume both premises are true, and the conclusion is false, we get a contradiction, i.e., something of the form p and not-p

Spoze the conclusion is false and both premises #1 and #2 are true. To say the conclusion is false, is to say not-A is false; so it is to say that A is true.
But since we are assuming premises #1 is true, then, together with A being true, we get that B is true.
But premises #2 asserts "not-B" which is to say B is false.
So we get B and not-B, i.e., B is true AND B is false which is a contradiction. *This proves the validity of modus tollens.*

I (A) Exercise (i). Prove the deductive validity of an argument with the following form:
1. Either A or B
2. Not B

Therefore A

The first premises, "either A or B" is going to be true so long as either A is true or B is true.
Premise #1 is called a <u>disjunction</u>, by the way, and this argument form is called disjunctive syllogism.

To prove the validity of an argument, you do NOT need to refer to any particular substitution for A and B, in fact you should not, because it has to be *perfectly general*. You have to show that whatever A and B are, an argument of this form can NEVER HAVE ALL TRUE PREMISES AND A FALSE CONCLUSION, without yielding a logical contradiction. Nevertheless, here's an example just to illustrate a case of an argument with this form (disjunctive syllogism):

Either Mark puts gas in his car or Mark's car eventually stops running.
Mark does not put gas in his case.

Therefore, Mark's car eventually stops running.

II. Valid arguments can have false conclusions.
Although a valid argument can never lead from all true premises to a false conclusion, THIS DOES NOT IMPLY THAT A VALID ARGUMENT ALWAYS HAS A TRUE CONCLUSION. Nor does it imply that if a valid argument has all false premises then it has a false conclusion. A valid argument can have all false premises and a true conclusion.

Exercise (ii) (I'll do this one) Give an example of an argument that follows the valid pattern of modus tollens that has a false conclusion. *Modus tollens* is

1. If A then B
2. Not-B

__
Therefore not A

To make "not-A" false (and this is the tricky part), you need a sentence A that is (clearly) true.

Example: Let A be: VT is in Virginia, and let B stand for: VT has the best football team in the country. Then the instance of modus tollens becomes:

1. If VT is in Virginia, then VT has the best football team in the country.
2. VT does not have the best football team in the country.

Therefore, VT is not in Virginia.

#1 is false, # 2 is true, and the conclusion is false.

II (B) Exercise (iii) (YOU do this one) Show that an argument following the form of a <u>disjunctive syllogism</u> in I (A) can have a false conclusion.

II (C). the bottom line is: a valid argument form can have EVERY COMBINATION of truth and falsity of its premises and conclusion except for <u>one</u>: It cannot have all true premises and a false conclusion. Therefore, if an argument is valid and the conclusion is false, you know that at least one of its premises must be false.

III. Invalidity
With an invalid argument form, it IS possible (logically) for all the premises to be true and yet the conclusion false. It is the DENIAL of a valid argument. *So long as you can dream up a particular instance of an argument that follows a given pattern that has all true premises and a false conclusion, then that argument is INVALID.*
So, unlike proving validity, to demonstrate invalidity, you do need to give a particular example, and for this purpose you want one where the conclusion is clearly false---i.e., one that the reader can see is false---and at the same time you want the premises to be clearly true.

III (A) Consider *affirming the consequent*:
1. If A then B
2. B

A

(the second premise asserts, and thus "affirms" the consequent of premise #1, that's why it is called this.)

To prove it is INVALID, start with a clearly false conclusion. A good example might be to choose a statement A that asserts some general claim, even though in fact that general claim is false, i.e., that it has exceptions. Then B could be a particular instance of the general claim.
For example:
A: VT won all the games it has ever played.
B: VT won a game against UVA on such and such a date.
Premise #1 is true, as is #2 (you fill in the date), but clearly the conclusion is false.

III (B) Now consider an invalid argument of the following form:
Either A or B.
Not-A
__
Therefore, not B

Exercise (iv) Prove the above argument is invalid by providing substitutions (i.e., sentences) for A and B such that the two premises are true and the conclusion is false. Again, remember, for a conclusion of this form to be false, not-B must be false, so B must be true.